# Review of the METS K-12 Mathematics Standards Core Concepts, Learning Goals and Performance Indicators

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## 1. Summary

Our overall impression of these standards is that, if adopted, they will not adequately prepare Missouri students either for studying mathematics at the college level or for using mathematics in other disciplines, particularly science and engineering. The proposed standards are grievously flawed in many ways. The standards, including the "Learning Goals and Performance Indicators" condemn our students to a second-rate education via the "tyranny of low expectations." They omit some essential topics or concepts altogether. Moreover, they are vague and filled with words and sentences devoid of mathematical content. As such, they provide little concrete direction to educators and make it impossible to assess whether a given student has attained them. Additionally, they are scatter-shot and have little depth.

In our detailed analysis of Learning Goals and Performance Indicators (LGPE) we have chosen to focus on Algebra I/Integrated I. The reasons for this choice are two-fold: We believe that the inadequacies found in the Algebra I/Integrated I standards are reflective of the problems found in the entire K-12 curriculum. Moreover, we only received a request to review this 76 page document in late May and felt that the best use of our limited time was to make a detailed analysis of a single, representative grade level. The fact that we do not make specific criticisms of the LGPE at other grade levels should in no way be conceived as an endorsement.

## 2. Algebra I

More than a quarter of the Core Concepts listed for Algebra I are devoted to data analysis. If this reflects a corresponding amount of class-time then students will be short-changed when it come to acquiring basic algebraic skills whose mastery is crucial for success in subsequent mathematics classes. Because students at this level do not have the necessary mathematical skills required for a deep understanding of elementary statistics we believe that data analysis should not be

emphasized. In particular, no more than 10% of class-time should be devoted to this topic.

Another general observation is that the standards for Algebra I are written with heavy emphasis on modeling and interpretation while the mastery of algebra techniques is left as a secondary goal. Take, for example, the Summary of Core Concepts. In the column Number/Algebra, the words *Interpret*, *Model*, *Represent*, are the key terms used and emphasized with *Solve* left for last. Our experience with incoming freshmen at MU underscores the fact that more and more students come to college lacking fluency in algebraic computations. Conceptual knowledge is important but students also need procedural fluency so that they can build on it in subsequent mathematics classes. Students for whom basic algebraic skills are not automatic will be handicapped in future mathematics classes.

The K-12 METS standards document with its emphasis on modeling and interpretation is also in stark contrast with the Major Topics of School Algebra as determined by the NMP's Task Group on Conceptual Knowledge and Skills, which refers to data in exactly one of the 27 topics listed (Section V.A). In addition, in the NMP report each topic is described in succinct terms - eg. "solving problems with linear equations" followed by a brief mathematical elaboration.

2.1. **Low expectations and lack of depth.** It is imperative that Missouri students be prepared to compete with students from across the country and around the world. In order to see how the expectations of the METS standards compare with those of other states, we chose those from two states with vibrant high-tech industries: California and Massachusetts. The proposed METS Algebra I standards expect far less of our students than either the California or Massachusetts Algebra I counterparts. Part of this may be attributed to the emphasis on data analysis, absent from the California standards and forming a much smaller portion of the Massachusetts standards. This has the effect of contributing to the problem of breadth without depth.

The California and Massachusetts standards include the topics of completing the square and the quadratic formula along with factoring as techniques for solving a quadratic equation. The K-12 METS standards omit this but includes some discussion of exponential functions. It is not clear how this can be done in any meaningful way given that there is apparently no discussion of rational exponents. Again, we see breadth ("study" more classes of functions) rather than depth (How do you solve a quadratic equation if you cannot factor it?).

In order to highlight the low expectations of the METS proposed standards, we include the following list of topics included in both the California and Massachusetts Algebra I standards, but postponed to Algebra II by METS. In compiling this list, we have erred on the side of omitting topics (for example, by attributing "Can graph a line, given an equation" to be covered by B1c: "Identify and translate among equivalent representations of linear expressions, equations, inequalities, and systems of equations, using verbal, tabular, graphical, and symbolic representations.") We have also paraphrased and condensed things substantially, so that several items in the California Mathematics Content Standards may be represented by a single item here.

**Specific topics included in California and Massachusetts Algebra I standards, but postponed to Algebra II by METS:**

- Add, subtract, and multiply polynomials. Divide polynomials by monomials.
- Simplify rational functions by finding the common factors and canceling.
- Solve quadratic equations (with real roots) by completing the square and by using the quadratic formula. Understand the relationship between these two methods.
- Solve equations and inequalities involving absolute value of linear expressions.
- Understand basic concepts of proof, and use properties of operations on real numbers to prove or disprove statements, identify errors in arguments, or justify each step in an argument. (This is actually omitted from the METS standards altogether, unless included implicitly somewhere.)

The California Mathematics Content Standards contain some subjects not included in the Massachusetts Learning Standards for Algebra I.

**Specific topics included in California Algebra I standards, but not in METS or Massachusetts [1]:**

- "Students apply quadratic equations to physical problems...."
- "Students add, subtract, multiply, and divide rational expressions and functions."
- "Students explain the difference between inductive and deductive reasoning and identify and provide examples of each."
- "Students identify the hypothesis and conclusion" and understand the purpose of counterexamples.

[1] All quotes here are from [1].

- "Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions."

The Massachusetts standards include a number of topics omitted by both California and METS.

2.2. **Important missing topics.** There are a number of crucial topics that are completely omitted from the Algebra I (and corresponding Integrated) Core Concepts, Learning Goals, and Performance Indicators. This does *not* include topics which many believe belong in Algebra I but are instead moved to Algebra II (see Section 2.1). It also does not include the many important skills which are not explicitly mentioned but which might be covered by the vague descriptions given. Also, we have made every effort (by, for example, searching the pdf file for related words) to determine whether these topics have been included in earlier grades. Any relevant success with these searches is mentioned here.

Here is an **incomplete** list of important omitted ideas or skills.

- *Function, domain and range.* Though students are expected to "Determine and justify whether a relationship is a function by using a graph or a verbal description" (A1a), there is no mention that students are expected to know the definition. Additionally, *domain* and *range* are completely omitted from the Algebra I standards. (They are mentioned in the Algebra II standards, but to understand the concept of a function, one needs to know the definitions of domain and range.)
- *Addition and subtraction of polynomials, or even of linear expressions or monomials.* We were unable to find any mention that students should be able to add and subtract polynomials, or even linear expressions or monomials, unless one of the following is supposed to include this:
  B1b) "Develop and justify equivalent algebraic expressions, equations, and inequalities using the properties of equality and inequality, as well as the commutative, associative, inverse, identity and distributive properties."
  B2b) "Describe and distinguish among the types of equations that can be constructed by equating linear expressions, including identities (e.g., $x + x = 2x$); equations for which there is no solution...."
- *Parallel and perpendicular lines.* While parallel and perpendicular lines are mentioned in the geometry standards, they are not included in the algebra standards. Students should know

the relationship between the slopes of two lines that are parallel (resp., perpendicular) and use this knowledge to write equations for lines parallel or perpendicular to a given line.

- *Multiplying monomials.* The LGPE include no mention that students should be able to actually multiply monomials or polynomials (other than linear ones), though they should "Compare (similarities and differences) multiplying numeric expressions versus multiplying algebraic expressions (e.g., binomial or trinomial expressions)" (C1c).
- *Roots, rational exponents, radicals.* We were unable to identify at what grade the definition of rational exponents is expected. Square (resp., cube) roots of perfect squares (resp. cubes) appear in grade 6, A3b, and from A3c "*Identify the two consecutive whole numbers between which the square root of a non-perfect square whole number is located" [2] we infer that square roots of whole numbers are to be discussed. Grade 8, Core Concept B has the title "Develop understanding of real numbers, roots, and exponents" though the word "root" appears nowhere else in the description or in the LGPE, nor is there any mention of fractional exponents. In fact, each use of the word exponent after the title is preceded by "positive- and negative-integer."

  The Algebra I LGPE include "evaluate the meaning of integer powers of variables in expressions, and apply the basic laws of exponents (e.g. $a^m \bullet a^n = a^{m+n}$)" (C2a). We understand both by inclusion of "integer powers of variables" and by the use of the letters "m" and "n" (traditionally used to denote integers) that the application of the laws of exponents is restricted to the case of integer exponents. In Algebra II, students are expected to "Extend the properties of rational exponents to real exponents, relating expressions with rational exponents to the corresponding radical expressions" (B1a). At what point are rational exponents or their properties to be introduced?
- *Simplify.* Surprisingly, there is no use of the word "simplify," though simplifying is an important algebraic skill.
- *Examples of contextual situations.* We counted at least eight uses of the word "contextual" in the 2 1/2 pages of the description of Algebra I (without the data analysis section). However, there is not a single example of a type of contextual situation which should be considered an important example. Compare

[2]Note that this is a "locally assessed item" which we may interpret to be considered less important.

this to the standards of both California and Massachusetts, which include "rate problems, work problems, and percent mixture problems" [1].

2.3. **Vague description and mathematical inaccuracies.** The mathematical standards one expects any state to adopt should be written using precise mathematical terms without nuance that leaves room for competing points of view. Each goal should be easily and unequivocally understood by educators. Unfortunately the current draft abounds with imprecise terminology which at times is mathematically inaccurate. Often detailed standards read like mathematical buzzwords strung together with the goal of impressing the public rather than precise benchmarks for the teachers who are meant to implement them. Here are **just a few** examples of inaccuracies or unclear statements.

- "Model, interpret, represent and solve nonlinear functions." (Summary of Core Concepts) One cannot solve a function.
- "Students identify situations with a constant rate of change..." (Core Concept A) A situation cannot have a rate of change. Same comment for " Use linear functions to interpret, model, and solve situations having a constant rate of change." (A2)
- "Analyze a contextual situation" (A2b) This is devoid of any mathematical meaning.
- "Identify and translate among equivalent representations of linear expressions, equations, inequalities, and systems of equations, using verbal, tabular, graphical, and symbolic representations." (B1c) How can one write an inequality or a system of equations using a table? In addition, a given table of points in the plane is not equivalent to a symbolic representation of a line without additional information.
- "Write, interpret, and translate among equivalent forms of linear equations and functions, including slope-intercept, point-slope, and general (standard) forms, recognizing that equivalent forms for a linear relationship reveal more or less information about a given situation." (B1c) If a linear relationship is written in two `equivalent` forms, one of these forms cannot reveal `more or less` information than the other.
- "Analyze and explain the reasoning used to solve linear equations and linear inequalities using symbolic methods, graphs, tables, and technology." (B3b) We are not aware of a way to solve linear equations and linear inequalities using tables. Just generating a table of randomly chosen values might create the impression that there is some type of trial and error approach

to solving linear equations and linear inequalities, which is far from reality. A similar comment applies to "Analyze and explain the reasoning used to solve a system of linear equations using graphs, tables, symbolic methods, and technology, and describe the nature of the solution (no solution, one solution, infinitely many solutions)." (B3d)

- "Provide and describe multiple representations of solutions to simple exponential and quadratic equations using manipulative models, verbal descriptions, tables, graphs, symbolic expressions, and technology." (C2d) We do not understand what the writers intended to single out here.

The standard Algebra I specifics take 2 1/2 pages (those related to data analysis fill an additional page). Despite this, there is almost no mention of specific skills that a successful Algebra I graduate should have. As an example, consider a few of the basic skills regarding lines and linear inequalities that one would expect a student finishing Algebra I to have:

- Graph a line, given an equation for the line.
- Sketch a region determined by a linear inequality.
- Given two distinct points on a line, or a point on the line and its slope, find an equation for the line.
- Find the slope and $x$- and $y$- intercepts of a line from its graph or from an equation for the line.

These specific skills appear to be included in

- (B1c) "Identify and translate among equivalent representations of linear expressions, equations, inequalities, and systems of equations, using verbal, tabular, graphical, and symbolic representations."
- (B1d) "Write, interpret, and translate among equivalent forms of linear equations and functions, including slope-intercept, point-slope, intercept, and standard forms, recognizing that equivalent forms for a linear relationship reveal more or less information about a given situation."

It is our opinion that the first list here (beginning "Graph a line...") gives clear direction to the reader. In contrast, the second list (taken from the K-12 METS standards) leaves room for guessing and requires a great deal of interpretation (in addition to containing mathematical inaccuracies).

## 3. Conclusion

A goal of the METS alliance is to ensure "that Missouri would continue to compete in the global market calling for increased skills in the areas of mathematics, engineering, technology, and science." We find it surprising, therefore, that the authors of these standards did not learn any lessons from those of states with thriving high-tech industries, such as California and Massachusetts.

In summary, the current version of the K-12 METS standards are flawed at multiple levels: they promote low standards in our classrooms, they omit critical topics, they provide little depth and they involve colloquial, imprecise language lacking mathematical clarity and specificity. They will not adequately prepare Missouri students either for studying at the college level or for using mathematics in other disciplines, particularly science and engineering. We strongly feel that they should be rewritten **from scratch** taking into account the recently released Final Report of the 2008 National Mathematics Advisory Panel. The K-12 METS standards should not be used as the working draft for a new document. Simple rearrangements or additions will not suffice to make this an acceptable document, at least for the upper grades. In addition, it is imperative that research mathematicians be an integral part of the committee charged with writing the K-12 mathematics standards. Adopting the METS proposed standards without doing this would be a disservice to the state of Missouri and will hurt its students for years to come.

## References

[1] *Algebra I Mathematics Content Standards*, California State Board of Education. Available at `http://www.cde.ca.gov/be/st/ss/mthalgebra1.asp`.

[2] *Mathematics Curriculum Framework, Learning Standards for Algebra I*, Massachusetts Department of Elementary and Secondary Education (November 2000). Available at `http://www.doe.mass.edu/frameworks/math/2000/algebra1.html`